Mathematical Approach for Shock Wave Problem:
A Case study of Hydrodynamic flow for two-dimensional dam-break problems

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Abstract

A numerical scheme of the two-dimensional shallow water equations for the dam break problem is presented. The methods are based on Godunov associated finite volume Roe's Riemann solver. Numerical results are presented for one- and two-dimensional dam break problems without source term present. For analytical test problem in one-dimensional has been derived by stoker (1957). Thus, for two-dimensional test problem we tested the same case from stoker, but we just expand the test cases on the y-direction.

Keywords: Two-dimensional shallow water equations, Finite volume method, Riemann solver, Roe's scheme, Dam break.

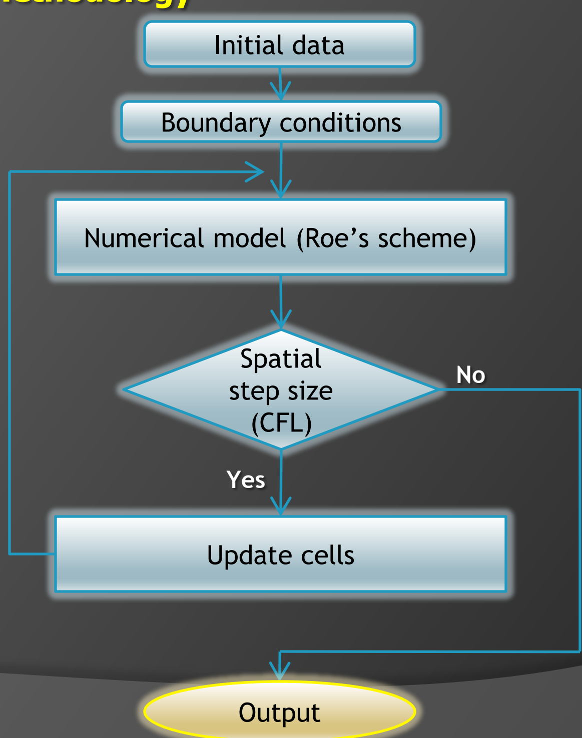
Introduction

The shallow water equations are a useful mathematical model for a good variety of fluid dynamics problems. Common examples are tidal waves in oceans, waves in shallow beaches, flood waves in rivers and dam-break wave modeling (Toro, 2001).

The most challenging feature of the shallow water wave equations is that they admit discontinuities and smooth solutions. Even the case in which the initial data is smooth can lead to discontinuous solutions in finite time. Such discontinuous often called 'shock' or 'shock waves', trigger the failure of a number of classical numerical methods.

In order to overcome the discontinuities problem due to shock, many numerical schemes were developed. One of them is numerical scheme based on the Gudanov approach which is aiming the best solution around the discontinuities. In this method, space is discretised into volumes, more often called cells, hence the general term of finite volume. We use finite volume methods based on Godunov associated approximate Riemann solver.

Methodology



Results

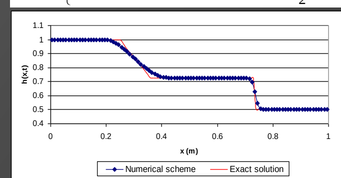
$$u(x,t)=\begin{cases} 0 & \text{if } x<\frac{1}{2}-t\sqrt{gh_L} \\ \frac{1}{3t}\left(2\left(x+t\sqrt{gh_L}\right)-1\right) & \text{if } \frac{1}{2}-t\sqrt{gh_L}\le x\le (u_2-c_2)t+\frac{1}{2} \\ u_2 & \text{if } (u_2-c_2)t+\frac{1}{2}<x\le St+\frac{1}{2} \\ 0 & \text{if } x>St+\frac{1}{2} \end{cases}$$

$$h(x,t)=\begin{cases} h_L & \text{if } x<\frac{1}{2}-t\sqrt{gh_L} \\ \frac{1}{9g}\left(2\sqrt{gh_L}-\frac{1}{2t}(2x-1)\right)^2 & \text{if } \frac{1}{2}-t\sqrt{gh_L}\le x\le (u_2-c_2)t+\frac{1}{2} \\ \frac{h_R}{2}\left(\sqrt{1+\frac{8S^2}{gh_R}}-1\right) & \text{if } (u_2-c_2)t+\frac{1}{2}<x\le St+\frac{1}{2} \\ h_R & \text{if } x>St+\frac{1}{2} \end{cases}$$

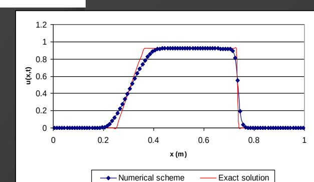
$$u_2=S-\frac{gh_R}{4S}\left(1+\sqrt{1+\frac{8S^2}{gh_R}}\right)$$

$$c_2=\sqrt{\frac{ghR}{2}\left(\sqrt{1+\frac{8S^2}{gh_R}}-1\right)}$$

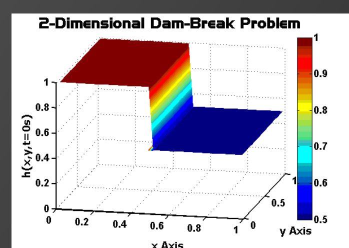
$$S=2.957918$$



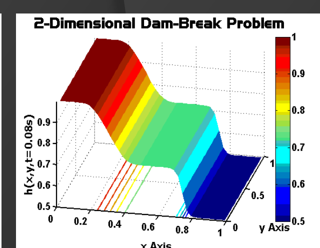
Comparison analytical and numerical(water depth(h))



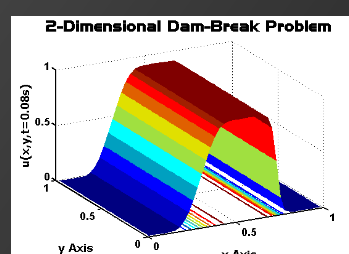
Comparison analytical and numerical (velocity (u))



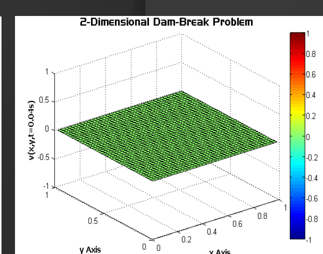
Initial condition



Water Depth (*h*)



Velocity in x-direction



Velocity in y-direction

Conclusions

The comparisons between analytical and numerical results for the 1D dam break problem have been shown, where the results could represent to two-dimensional in the same cases. Finite volume method provides accurate results. On this case, there are velocity in x-direction, but in y-direction is zero.

From the numerical schemes and results are presented above, we can conclude that the Roe's scheme is very powerful to solve hydrodynamic problem when the shock s waves are existed.

References

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